Table 1 Typical control loader forces

		Force, lb	
n_x	n_z	Total	Maneuver increment
0	1	0.58	0
0	3	0.73	0.15
0.5	1	3.09	2.52

The effect of the loader on pilot perception was tested in a series of simulated "flights" piloted by Capt. Mark Jackson of the U.S. Army. The scenario was selected so as to induce significant flight loads. It consisted of takeoff, vertical climb to 50 ft, rapid acceleration to 80 knots, a 180-deg turn to the right at a bank of at least 60 deg, return to the airport, rapid deceleration to hover, and landing. This typically lasted 3 min and 15 s. The pilot elected to fly with the trim off.

The flight sequence was practiced with an early version of the control model with which the pilot was already familiar. Once proficiency was achieved, six flights were made with five variations in the loader program: 1) early version (gain factors not at their correct final values), 2) full model, 3) full model except that gain for control acceleration was set to zero, 4) full model except that gain for flight acceleration was set to zero, and 5) dummy, control loader applying no force.

The order of the six flights was 1, 3, 2, 5, 4, 2. The pilot did not know what each run represented. He was asked to rate the control feel subjectively. The results were 1) good; 2) great, good; 3) good; 4) good; and 5) poor. The full loader model 2 was included twice. Not surprisingly, it got two different ratings in the subjective evaluation. Still, the full model rated higher than any other.

Table 1 presents typical values of commanded force at the cyclic grip. The control loader produces a force also when the cyclic stick is accelerated; 1 g at the grip gives rise to 0.50 lb from the loader. This makes up for the difference in linkage inertia between simulator and aircraft. For further details of the trial implementation see Ref. 6.

V. Conclusions

The new method requires more detailed data on the control system than the traditional and has the capability of delivering higher fidelity. The contemporary McFadden digital loader can support the new method. The effort of modeling and programming is moderate.

Acknowledgments

This work was supported by Grant 92G0012 from the Federal Aviation Administration, which also covered 50% of the cost of the control loader. The balance was borne by the James R. Cudworth Endowment at the University of Alabama. We thank Jack Dohme and William Barker of the U.S. Army Research Institute for access to data on the UH1. The help of K. Krishnakumar in securing data is appreciated. We are indebted to Mark Jackson of the U.S. Army who flew the tests. We also thank Melissa Evans who served as the FDL monitor at the time of the tests.

References

[1]Rolfe, J. M., and Staples, K. J., *Flight Simulation*, Cambridge Univ. Press, Cambridge, England, UK, 1986, pp. 77–81.

[2]Anon., "Airplane Simulator Qualification," U.S. Dept. of Transportation, Federal Aviation Administration, Advisory Circular 120-40B, July 1991, Appendix 3, pp. 18, 19.

[3]Anon., "Airplane Flight Simulator Evaluation Handbook," International Standard for the Qualification of Airplane Flight Simulators, 1st ed., London, March 1993, pp. 2B-1, 2B-2.

[4]Anon., "International Standard for the Qualification of Airplane Flight Simulators," The Royal Aeronautical Society, London, Jan. 1992, Appendix 1, p. 5.

[5]Anon., "Instruction Manual: Universal Variable Digital Cockpit Control Force Loading System, Model 192B," McFadden Systems, Inc., Santa Fe Springs, 1993.

[6]Katz, A., and Schamlé, M., "Methodology for Integration of Digital Control Loaders in Aircraft Simulators," *Proceedings of the AIAA Simulation Technologies Conference* (Monterey, CA), AIAA, Washington, DC, 1993, pp. 36–43 (AIAA Paper 93-3551).

[7]Etkin, B., *Dynamics of Flight—Stability and Control*, Wiley, New York, 2nd ed., 1982, p. 94.

[8]Anon., "Banshee System Instruction Manual," Atlanta Signal Processors, Inc., Atlanta, GA, 1990.

Improved Literal Approximation for Lateral-Directional Dynamics of Rigid Aircraft

Rafael Livneh*

Arizona State University, Tempe, Arizona 85287-8006

Introduction

THE literal (analytical) approximations for the spiral, roll convergence, and dutch roll modes are traditionally obtained by discarding some of the dynamic equations and certain degrees of freedom associated with the lateral-directional dynamics of a rigid airplane. In most cases, this leads to an "overdecoupling" of the lateral-directional dynamics, resulting in approximations that are rather inaccurate for both the spiral eigenvalue λ_s and the dutch roll damping $2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}$ and are at best reasonable for the roll convergence eigenvalue λ_r and the dutch roll frequency $\omega_{n_{\mathrm{DR}}}^2$. In this Note, the literal approximation method developed by Newman and Schmidt[1] and Livneh and Schmidt[2] is used to generate one set of unified approximations for the lateral-directional dynamics of six different rigid airplanes with a total of 16 sets of flight conditions.[3,4] Various observations regarding the interrelations between the unified and the traditional approximations are drawn, and a short analysis of the errors associated with the unified vs the literal approximations is provided.

Formulation and Traditional Approximations for Lateral-Directional Dynamics

The characteristic polynomial corresponding to the Laplace transform of the linearized equations of motion for the lateral-directional dynamics of a rigid airplane is given[4] by

$$D(s) = \det\begin{bmatrix} sU_1 - Y_\beta & -[g\cos(\theta_1) + sY_p] & s(U_1 - Y_r) \\ -L_\beta & s^2 - L_p s & -\left(s^2 A_1 + sL_r\right) \\ -N_\beta & -\left[s^2 B_1 + N_p s\right] & s^2 - sN_r \end{bmatrix}$$

$$= (1 - A_1 B_1)U_1 s(s+\lambda_s)(s+\lambda_r)\left(s^2 + 2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}} s + \omega_{n_{\mathrm{DR}}}^2\right) = 0 \tag{1}$$

where U_1 is the flight vehicle airspeed, g is the acceleration of gravity, θ_1 is the pitch angle, and s is the Laplace transform variable. The three triads $(Y_\beta, L_\beta, N_\beta)$, (Y_p, L_p, N_p), and (Y_r, L_r, N_r) correspond to the side acceleration, rolling moment, and yawing moment per the perturbed sideslip angle β, perturbed roll rate p, and perturbed yaw rate r, respectively. The dimensionless inertia quantities A_1 and B_1 are given by $A_1 \equiv I_{xz}/I_{xx}$ and $B_1 \equiv I_{xz}/I_{zz}$, where I_{xx} and I_{zz} are the moments of inertia about the X and Z stability axes, respectively, and I_{xz} is the product of inertia about the X, Z

Received July 25, 1994; revision received March 2, 1995; accepted for publication March 3, 1995.

*Research Scientist, Aerospace Research Center. Member AIAA.

stability axes. The traditional lateral-directional approximations can be found in most textbooks on flight dynamics[4–7] to be

$$\bar{\lambda}_s = \frac{L_\beta N_r - L_r N_\beta}{L_\beta}, \qquad \bar{\lambda}_r = -L_p$$
$$\overline{\omega^2_{n_{\mathrm{DR}}}} = N_\beta + \frac{N_r Y_\beta - N_\beta Y_r}{U_1}, \qquad \overline{2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}} = -N_r - \frac{Y_\beta}{U_1} \tag{2}$$

It is well known, and also quite apparent from Table 1, that these approximations are rather inaccurate for both λ_s and $2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}$ and are at best reasonable for λ_r and $\omega^2_{n_{\mathrm{DR}}}$. This has led authors such as Roskam[4] to altogether discourage the use of the dutch roll approximation for practical applications. The results reported here aim at improving on the traditional approximations (2) by using the literal approximation method.[1–3]

Literal Approximations for Lateral-Directional Dynamics

The literal approximation method seeks to obtain approximate literal (analytical) expressions for the roots of a given polynomial in terms of its coefficients by using a first-order sensitivity approach. For the case at hand, the coefficients of $D(s)$ are approximated by a first-order Taylor series expansion about the coefficients of a user-prespecified nominal polynomial $\bar{D}(s)$ with known nominal roots. The resulting approximations are then used to obtain a literal expansion of the actual polynomial roots about the nominal literal roots, yielding the full literal approximations for the roots of $D(s)$. The reduced literal approximations for a given aircraft configuration are obtained by retaining only the numerically dominant literal terms in the full approximations. Different airplanes might result in different reduced approximations due to retention of different dominant literal terms during the approximation process. When such differences occur, one can use the union of the differing reduced approximations to obtain a set of unified literal approximations.

Following the derivation associated with the literal approximation method,[1,2] the nominal values associated with the spiral mode, the roll convergence mode, the dutch roll damping, and the dutch roll natural frequency are given by $\bar{\bar{\lambda}}_s = 0, \bar{\bar{\lambda}}_r = \bar{\lambda}, \overline{\overline{2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}}} = \overline{2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}}$, and $\overline{\overline{\omega^2_{n_{\mathrm{DR}}}}} = \overline{\omega^2_{n_{\mathrm{DR}}}}$, respectively. In this Note a threshold level of $p = 10\%$ was chosen to discard insignificant terms, and reduced literal approximations were independently derived[3] for the six airplanes in 16 flight conditions tabulated in Appendix C of Roskam.[4] The unified literal approximations for the lateral-directional dynamics of these airplanes are given by

$$\lambda_s \cong \frac{L_\beta g\cos(\theta_1)}{(1 - A_1 B_1)U_1\bar{\lambda}_r\overline{\omega^2_{n_{\mathrm{DR}}}}}\bar{\lambda}_s$$

$$\lambda_r \cong \bar{\lambda}_r + L_\beta\frac{N_p - B_1\bar{\lambda}_r - g\cos(\theta_1)/U_1}{(1 - A_1 B_1)\left(\bar{\lambda}_r^2 - \overline{2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}}\,\bar{\lambda}_r + \overline{\omega^2_{n_{\mathrm{DR}}}}\right)}$$

$$\omega^2_{n_{\mathrm{DR}}} \cong \overline{\omega^2_{n_{\mathrm{DR}}}} + L_\beta\frac{B_1\overline{\omega^2_{n_{\mathrm{DR}}}} + \bar{\lambda}_r\{N_p^- g\cos(\theta_1)/U_1\}}{(1 - A_1 B_1)\left(\bar{\lambda}_r^2 - \overline{2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}}\,\bar{\lambda}_r + \overline{\omega^2_{n_{\mathrm{DR}}}}\right)} \tag{3}$$

$$2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}} \cong \overline{2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}} - L_\beta\frac{N_p - B_1\bar{\lambda}_r g\cos(\theta_1)/U_1}{(1 - A_1 B_1)\left(\bar{\lambda}_r^2 - \overline{2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}}\,\bar{\lambda}_r + \overline{\omega^2_{n_{\mathrm{DR}}}}\right)}$$

The evaluation of the unified approximations given by Eq. (3) can be thought of as a two-tier process consisting of a) evaluation of the traditional approximations given by Eq. (2) and b) evaluation of the unified approximations given by Eq. (3). It is noted that the correction term to λ_r is equal to the negative of the correction to $2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}$.

Enhanced Dynamic Coupling in Unified Approximations

The traditional spiral mode approximation is based on the assumption that the forces associated with the roll angle rate $\dot{\phi}$ are small compared to the forces associated with the sideslip angle β and the heading angle rate $\dot{\psi}$. The side-force equation and the roll angle ϕ degree of freedom are therefore eliminated from the system, leaving the stability derivatives associated with β and heading angle ψ, namely L_β, L_r, N_β, and N_r, as the only factors affecting λ_s. The unified approximations of Eq. (3) recapture some of the dependence of λ_s on both ϕ and the side-force equation. The traditional dependence of λ_s on L_β, L_r, N_β, and N_r is supplemented by L_p, Y_β, and Y_r (through $\bar{\lambda}_r$ and $\overline{\omega^2_{n_{\mathrm{DR}}}}$), associated with both ϕ and the side-force equation. The unified approximation for λ_s also includes explicit dependence on θ_1, U_1, and the inertia properties A_1 and B_1 of the aircraft. The unified approximations for λ_r, $2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}$ and $\omega^2_{n_{\mathrm{DR}}}$ recapture some of the dependence on the degrees of freedom and the force and moment equations discarded in the traditional approximations. The traditional dependence of λ_r on L_p is supplemented by further dependence on $Y_\beta, L_\beta, N_\beta, Y_r, N_r, N_p, \theta_1, U_1, A_1$, and B_1. The unified approximations for $\omega^2_{n_{\mathrm{DR}}}$ and $2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}$ recapture some of the coupling between the dutch roll and both ϕ and the rolling moment equation by supplementing the traditional dependence of $\omega^2_{n_{\mathrm{DR}}}$ and $2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}$ on $N_\beta, N_r, Y_\beta, Y_r$, and U_1, with L_β, L_p, N_p, A_1, and B_1. In fact, $\lambda_s, \lambda_r, 2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}$, and $\omega^2_{n_{\mathrm{DR}}}$, which were dependent on 4, 1, 5, and 3 parameters, respectively, in the traditional approximations, are each dependent on 12 airplane/flight condition parameters for the unified approximations.

Relative Accuracy of Unified vs Traditional Approximations

Table 1 is an example of the literal approximation error analysis carried out for the supersonic fighter bomber airplane representative of the McDonnell Douglas F4C.[4] Similar analyses were conducted[3] for the other five configurations in Appendix C of Roskam.[4] The three clusters of four rows in Table 1 correspond to three distinct flight conditions, namely power approach, subsonic cruise, and supersonic cruise. The second column (from the left) lists the exact numerical value of the lateral-directional parameters in question. These values correspond to the roots of $D(s)$ evaluated for the stability derivatives and flight conditions of the McDonnell Douglas F4C.[4] The third column provides the numerical value of the lateral-directional parameters obtained by using the traditional approximations of Eq. (2), whereas the fourth column lists the percentage error between the traditional approximations for these parameters and their exact values. The fifth and sixth columns provide the numerical values and errors associated with the full literal approximations, whereas the seventh and eighth columns provide the numerical values and errors associated with the unified literal approximations of Eqs. (3). The full approximation error gives an indication of the improvement potential of the literal approximation method vs the traditional approximations and exhibited improvement over the traditional approximation for all the cases tested. The unified approximations resulted in improved accuracy for all but the power approach roll convergence mode of Table 1, where the traditional approximation is 12.8% more accurate than the unified approximation.

It was found that the accuracy of both the full and unified approximations increased significantly for cruise with $U_1 \geq 250$ ft/s. This led to the construction of Table 2, where the two accuracy measures used are a) range of the absolute value of the error and b) range of the absolute values of the ratios of the traditional error vs either the full or the unified error. The range of the absolute value of the error is obtained by computing the smallest and the largest percentage error for the quantity in question. Based on the three cases presented in Table 1, for example, the range of the absolute values of the error in the unified approximation for λ_r is 1.5–28.5%. A small range with a low upper value of the approximation error is, of course, indicative of a good approximation. The range of the absolute values of the error ratio is obtained by dividing the traditional error by either the full or the unified error and finding the largest and the smallest absolute values of these ratios among the cases examined. For example, the range of the absolute value of the error ratio for the full approximation of $2\zeta_{\mathrm{DR}}\omega_{n_{\mathrm{DR}}}$

Table 1 Comparative accuracy analysis between traditional, full, and unified literal approximations, for supersonic fighter bomber airplane representative of McDonnell Douglas F4C[a]

Parameter	Exact value	Traditional approximation Value	Traditional approximation % Error	Full approximation Value	Full approximation % Error	Unified approximation Value	Unified approximation % Error
			Power approach				
λ_s	0.01998	0.04891	144.8	0.02635	31.9	0.02635	31.9
λ_r	0.9251	1.07	15.7	0.9031	−2.4	0.6616	−28.5
$\omega^2_{n\mathrm{DR}}$	3.054	2.002	−34.4	2.967	−2.9	3.041	−0.4
$2\zeta_{\mathrm{DR}}\omega_{n\mathrm{DR}}$	0.894	0.3596	−59.8	0.9096	1.7	0.768	−14.1
			Subsonic cruise				
λ_s	0.01313	0.1476	1025	0.0157	19.6	0.0157	19.6
λ_r	1.339	1.231	−8.0	1.349	0.7	1.359	1.5
$\omega^2_{n\mathrm{DR}}$	5.748	5.225	−9.0	5.743	0.0	5.811	1.1
$2\zeta_{\mathrm{DR}}\omega_{n\mathrm{DR}}$	0.2312	0.3444	49.0	0.2186	−5.4	0.2168	−6.2
			Supersonic cruise				
λ_s	0.04951	0.1745	252.4	0.0632	27.6	0.0632	27.6
λ_r	0.6797	0.7897	16.2	0.6411	−5.7	0.6962	2.4
$\omega^2_{n\mathrm{DR}}$	8.556	5.769	−32.6	8.523	−0.4	8.612	0.7
$2\zeta_{\mathrm{DR}}\omega_{n\mathrm{DR}}$	0.3321	0.2612	−21.3	0.357	7.5	0.3547	6.8

[a]From Ref. 3.

Table 2 Comparison between traditional, full, and unified approximations using measures of range of error and range of ratio of errors for the cruise configurations with $U_1 \geq 250$ ft/s

Parameter	Range of absolute value of error, % Traditional	Full	Unified	Range of factor of improvement of traditional vs new approximation error Full	Unified
λ_s	252–2879	0.1–28	0.1–28	9.1–13,890	9.1–13,890
λ_r	1.4–16.2	0.0–5.7	0.1–4.8	7–304	3.7–30
$\omega^2_{n\mathrm{DR}}$	0.5–33	0.0–4.3	0.2–6.3	1.2–284	1.6–46
$2\zeta_{\mathrm{DR}}\omega_{n\mathrm{DR}}$	10.8–521	0.3–39	0.0–39	2.8–180	1.3–1360

corresponding to the three cases listed in Table 1 is obtained by comparing the ratios $|(-59.8)/1.7| = 35.2$, $|49.0/(-5.4)| = 9.1$, and $|-21.3/7.5| = 2.8$. The range of the absolute values of the ratio of the error is therefore 2.8–35.2. A wide range with a large lower value of the ratio of the approximation error is indicative of a significant improvement of either the full or the unified approximation over the traditional approximations. Since the traditional approximation is known to yield reasonable results for λ_r and $\omega^2_{n\mathrm{DR}}$, the most improved approximations are for λ_s and $2\zeta_{\mathrm{DR}}\omega_{n\mathrm{DR}}$. The improvement factor of up to 13,890 in λ_s is due in part to its proximity to the origin. For all the airplanes analyzed, the approximation for λ_s is at least 143% accurate for the combined cruise and approach flight conditions and at least 28% accurate for the cruise conditions with $U_1 \geq 250$ ft/s. Both the full and the unified approximations for $2\zeta_{\mathrm{DR}}\omega_{n\mathrm{DR}}$ exhibited errors of at most 40% vs maximal error of 521% in the traditional approximations. The error in the approximation for λ_r was improved from at most 49% (traditional) to at most 28% (unified) for the combined cruise and approach flight conditions and by at most 16.2% (traditional) to at most 4.8% (unified) for the cruise conditions. The error in the approximation for $\omega^2_{n\mathrm{DR}}$ was reduced from 50% (traditional) to 25% (unified) for both cruise and approach flight conditions and from 33% (traditional) to 6.3% (unified) for the cruise flight conditions.

Conclusions and Recommendations

The literal approximation method was successfully applied to generate unified literal approximations for six different airplanes flying at a total of 16 different flight conditions. The unified approximations exhibited meaningful improvement in accuracy vs the traditional approximations at the price of a very manageable increase in the complexity of the formulation. Most improved were the approximations for the spiral eigenvalue λ_s and the dutch roll damping $2\zeta_{\mathrm{DR}}\omega_{n\mathrm{DR}}$. The unified approximations supplement the traditional approximations with additional cause-and-effect relations between the airplane/flight condition properties and its lateral-directional stability dynamic characteristics by capturing more of both the coupling between the spiral roll convergence and the dutch roll modes and the dependence of these modes on the stability derivatives, mass properties, and flight conditions of the airplane.

Acknowledgment

The author thanks Karl D. Bilimoria for his helpful insights and valuable technical comments during the course of this work.

References

[1]Newman, B., and Schmidt, D. K., "Numerical and Literal Aeroelastic Vehicle-Model Reduction for Feedback Control Synthesis," *Journal of Guidance, Control, and Dynamics*, Vol. 14, No. 5, 1991, pp. 943–953.

[2]Livneh, R., and Schmidt, D. K., "New Literal Approximations for the Longitudinal Dynamic Characteristics of Flexible Flight vehicles," *Proceedings of the AIAA Guidance, Navigation, and Control Conference* (Hilton Head, SC), AIAA, Washington, DC, 1992, pp. 536–545 (AIAA Paper 92-4411).

[3]Livneh, R., "Improved Lateral-Directional Approximations for Rigid Aircraft Using the Literal Approximation Method," *Proceedings of the AIAA Guidance, Navigation, and Control Conference* (Baltimore, MD), AIAA, Washington, DC, 1995 (AIAA Paper 95-3307) (to be published).

[4]Roskam, J., *Airplane Flight Dynamics and Automatic Flight Control*, Roskam Aviation and Engineering Corp., Ottawa, KS, 1982.

[5]Perkins, C. D., and Hage, R. E., *Airplane Performance Stability and Control*, Wiley, New York, 1949.

[6]McRuer, D., Ashkenas, I., and Graham, D., *Aircraft Dynamics and Automatic Control*, Princeton Univ. Press, Princeton, NJ, 1973.

[7]Nelson, R. C., *Flight Stability and Automatic Control*, McGraw–Hill, New York, 1989.